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Perspective

Why the Simplex Method Still Obscures the Merits of the Face Method in Scarce-Resource Allocation

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Abstract

This article examines why the Simplex Method continues to dominate scarce-resource allocation despite the development of the Face Method as an alternative approach to linear programming. It argues that simplex dominance derives not necessarily from universal computational superiority, but from nearly eight decades of accumulated research, pedagogical familiarity, mature software infrastructure, extensive practical applications, and well-developed sensitivity and post-optimality analysis. In contrast to the vertex-to-vertex progression associated with simplex algorithms, the Face Method proposes movement from face to face within the feasible polyhedron, with the objective of reaching an optimal face and solution. Although the Face Algorithm predates the 2025 monograph by Pan, subsequent work has examined its implementation and numerical performance. These studies provide a basis for further investigation, but they do not by themselves establish universal superiority over established methods. The article therefore reviews the historical and computational context of the Face Method and advocates rigorous independent benchmarking of Simplex, Face, and interior-point methods across diverse scarce-resource allocation problems. Such evaluations should consider computational speed, numerical stability, scalability, memory requirements, reoptimization, sensitivity analysis, and interpretability. It concludes that the Face Method represents a significant research direction whose broader adoption will depend on the strength, reproducibility, and practical relevance of comparative evidence.

Introduction

Scarcity is at the heart of economics, management science, and operations research. Firms, governments, and individuals routinely confront the problem of allocating limited labour, capital, materials, machine hours, energy, and other resources among competing activities. Linear programming (LP) provides one of the most influential mathematical frameworks for addressing this problem, and for nearly eight decades the Simplex Method has been its most recognizable computational technique. Developed by George B. Dantzig in 1947, the Simplex Method transformed constrained optimization from an abstract mathematical problem into a practical instrument for planning and decision-making [1, 2]. Its influence on optimization and economic analysis has consequently been enormous [3, 4]. Yet longevity should not be mistaken for perfection. The Simplex Method has known theoretical weaknesses. Most notably, particular pivoting rules can exhibit exponential worst-case behaviour, as demonstrated by Klee and Minty [5]. This and other limitations stimulated decades of research into alternative approaches, including polynomial-time algorithms and interior-point

methods [6]. The Face Algorithm, developed in earlier work by Pan and subsequently studied by other researchers, offers another approach to linear programming. Zhang, Yang, and Liao [7], for example, examined an efficient implementation of the Face Algorithm and compared its numerical performance with the Simplex Method. Pan's 2025 monograph later brought together the Face Method and several of its variants in a comprehensive treatment [8]. The development of the Face Method therefore raises an important question: Why does the Simplex Method continue to overshadow the Face Method and other alternative optimization algorithms in scarce-resource allocation? The answer, in my view, lies less in proof that simplex is universally computationally superior and more in history, accumulated implementation experience, pedagogical familiarity, software infrastructure, interpretability, and the formidable advantage enjoyed by an incumbent technology. The Simplex Method currently dominates because it has had almost eighty years to become an ecosystem; the Face Method, although not entirely new, is still developing the broader body of independent evidence and practical adoption needed to challenge that ecosystem.

Discussion

Simplex Has the Advantage of an Extraordinary Historical Head Start

The first explanation is straightforward. The Simplex Method has existed since 1947, whereas the Face Method has a shorter history of development and dissemination as a distinct framework. The Face Algorithm was already the subject of computational research before Pan's 2025 monograph. Zhang et al. [7] investigated an improved implementation, while Pan (2015) subsequently published a response addressing aspects of that implementation. Pan's monograph, Face Method: Toward the New Generation of Linear Programming, published in 2025, provides a more comprehensive presentation of the basic Face Method together with generalized, dual, reduced, and Cholesky-factorization variants [8]. This difference in historical development matters. An optimization algorithm does not become dominant merely because its mathematical foundations are sound. It must survive repeated applications, computational experiments, modifications, criticism, teaching, software implementation, and comparison with competitors. Simplex has passed through all these stages. It has been applied to production planning, transportation, scheduling, blending, finance, energy systems, logistics, agricultural planning, and numerous other resource-allocation problems. Dantzig [2] described linear programming as enabling optimal policy decisions for broad classes of complex practical problems. Later assessments similarly emphasize the extraordinary influence of simplex-based LP on operations research and computational optimization [4]. The Face Method has not yet accumulated a comparable breadth of software adoption and application history. Nevertheless, it would be inaccurate to describe it as a method introduced only in 2025. Its earlier development and the subsequent implementation studies should be recognized when assessing its present position. Consequently, comparing the current visibility of the two approaches is not a comparison between technologies that have competed under equivalent historical conditions. It is a comparison between an established institution and a developing alternative.

Simplex Is Embedded in the Language of Resource Allocation

A second advantage is pedagogical. Generations of students in operations research, management science, economics, engineering, accounting, and business administration have learned scarce-resource allocation through the familiar sequence of decision variables, objective functions, constraints, slack variables, tableaux, entering variables, leaving variables, pivot operations, and optimal solutions. This, vocabulary has become almost synonymous with linear programming. More importantly, the economic interpretation of an LP solution is particularly valuable in resource-allocation decisions. A decision-maker normally wants more than an optimal quantity of products. Management may also need to know which resources are fully utilized, which constraints are nonbinding, what slack remains, how much an additional unit of a scarce resource is worth, and whether changes in objective-function coefficients would alter the optimal allocation. The primal-dual structure underlying LP provides the mathematical basis for this kind of economic interpretation. Thus, the dominance of simplex is partly cognitive. Managers, teachers, analysts, textbook writers, and software developers already understand its language. A challenger must demonstrate sufficiently substantial advantages to justify the intellectual and institutional cost of switching. Familiarity alone does not prove computational superiority, but it can influence which methods are taught, implemented, and selected in practice.

Mature Software Gives Simplex an Enormous Practical Advantage

The third explanation is computational infrastructure. The modern Simplex Method is not merely Dantzig's original algorithm frozen in 1947. Decades of research have

produced revised-simplex procedures, improved pivot strategies, presolve techniques, sparse-matrix methods, numerical-stability improvements, basis factorization and refactorization procedures, scaling methods, and sophisticated warm-start mechanisms. Bixby [10], for example, documented enormous advances in the ability of optimization software to solve real-world LP problems. Such progress demonstrates an important principle in algorithmic competition: mature implementation can matter as much as theoretical elegance.

The simplex algorithm also illustrates the distinction between worst-case theoretical complexity and observed computational behaviour. Klee and Minty [5] demonstrated that particular simplex pivoting rules can follow exponentially long paths on specially constructed problems. Yet practical implementations have remained highly effective on many real-world LP instances [4, 10]. The development of polynomial-time alternatives did not make simplex obsolete. Karmarkar's [6] interior-point breakthrough dramatically changed LP theory and computation, but simplex algorithms continued to occupy an important position in optimization software. This historical experience should also caution against premature conclusions about the Face Method. The 2013 study by Zhang et al [7]. reported favourable numerical performance for the Face Algorithm in the computational setting they investigated and proposed an implementation with lower computational cost per iteration and favourable properties for sparse problems. These findings are relevant, but they should be interpreted within the scope of that study rather than treated as proof of general superiority across all LP applications [7]. Theoretical shortcomings of simplex alone are insufficient to establish the practical superiority of an alternative. The Face Method must ultimately be evaluated through reproducible computational benchmarking on diverse classes of real resource-allocation problems.

The Face Method Challenges a Fundamental Simplex Convention

The Face Method deserves serious attention because it questions the necessity of the vertex-to-vertex movement that characterizes the conventional geometric interpretation of simplex. The feasible region of an LP problem is a convex polyhedron. In the usual geometric interpretation, simplex moves through adjacent basic feasible solutions, which correspond to vertices, until an optimal solution is reached. The Face Algorithm follows a different strategy: it moves from face to face of the feasible polyhedron with the objective of reaching an optimal face and solution. Zhang et al. [7] describe the search direction in terms of the orthogonal projection of the negative objective gradient onto a relevant null space. Their work also examines an improved implementation of the algorithm. Pan's 2025 monograph develops the basic method and several extensions, including generalized, dual, reduced, and Cholesky-factorization versions [8, 7]. Conceptually, this is significant. A face can contain multiple points and, in some cases, multiple optimal solutions. An approach directed toward an optimal face therefore differs from one whose iterates are restricted to vertices. The distinction is not simply that one method moves farther at each step; the methods use different geometric and computational procedures. The possible value of the Face Method must be assessed in relation to the structure of the problem, the computational work required at each iteration, and the numerical behaviour of the implementation. However, conceptual attractiveness is only the beginning of the argument. In scarce-resource allocation, the decisive questions are empirical: Does the method solve realistic problems faster? Does it require less memory? How does it behave under degeneracy? Is it numerically stable? Can solutions be efficiently reoptimized after resource availability changes? Can it exploit sparsity? How easily can shadow-price and sensitivity information be extracted? And does its performance advantage, if any, increase as problem dimensions grow? The existing implementation research provides a useful starting point, but further independent studies are needed to answer these questions across a wider range of problem classes. A small number of favourable computational results should encourage further investigation, not replace it. Until a broader body of evidence answers these questions, practitioners have understandable reasons to remain with established algorithms.

Scarce-Resource Decisions Require Interpretability, Not Merely an Optimum

This issue is particularly important in management applications. Consider a manufacturer allocating labour hours, raw materials, machine capacity, and working capital among several products. Obtaining the maximum contribution margin is only one part of the decision. Management may subsequently ask: Which resource is actually scarce? What is the opportunity cost of one additional labour hour? How much unused material remains? Within what range can resource availability change without altering the current optimal basis? Which product would become attractive if its contribution margin increased?

The long development of simplex-based sensitivity analysis makes these questions relatively natural to answer in established LP software. Shadow prices, reduced costs, binding constraints, allowable increases and decreases, and post-optimality analysis are deeply integrated into LP teaching and software. These concepts arise from the primal-dual structure of linear programming and are not exclusive to the Simplex Method. However, the availability, presentation, and practical convenience of equivalent sensitivity information can differ between algorithms and software implementations. For the Face Method to become equally attractive for scarce-resource allocation, its literature and implementations will need to demonstrate how corresponding managerial information can be obtained and interpreted. This includes explaining how dual information, shadow prices, reduced costs, and changes in resource availability can be evaluated within the face-based framework. The present literature cited here does not establish that the Face Method lacks these capabilities; rather, their practical accessibility and comparability with mature simplex-based sensitivity tools require clearer documentation and further evaluation. A mathematically sophisticated algorithm can remain marginal in applied management if decision-makers cannot easily translate its computations into economic meaning.

Interior-Point Methods and the Continuing Competition Among Algorithms

The comparison between simplex and the Face Method should also be placed in the wider history of linear programming algorithms. Karmarkar's [6] polynomial-time algorithm helped establish the importance of interior-point approaches, which use a different computational strategy from the boundary-based progression associated with simplex and face algorithms. Interior-point methods became an important alternative, particularly for certain large-scale LP problems, and their development contributed substantially to modern optimization. However, the emergence of interior-point methods did not eliminate simplex from practical use. The two approaches have different computational characteristics, and their relative performance depends on problem structure, numerical requirements, software implementation, and the need for information such as a basic solution or a warm start. Warm-start capabilities should not be treated as an absolute limitation of every interior-point implementation; they depend on the method and the implementation. Similarly, interior-point methods can be used to solve LP relaxations within integer-programming procedures, even though solving an LP relaxation is not the same as solving the complete integer problem. This history is relevant to the Face Method. It shows that introducing a theoretically distinct algorithm does not automatically displace an established approach. Instead, methods may coexist because different problem structures and practical requirements favour different computational strategies. The appropriate comparison is therefore not whether one algorithm is universally best, but which methods perform most effectively for particular classes of problems and decision-making needs.

Incumbency Should Not Become Intellectual Complacency

None of this implies that the Simplex Method should remain permanently dominant. Indeed, the history of optimization warns against confusing familiarity with optimality. Karmarkar's [6] polynomial-time algorithm showed how a fundamentally different computational perspective could transform LP research, while subsequent improvements in both interior-point and simplex implementations demonstrated that competition among algorithms can strengthen the entire field. The same openness should be extended to the Face Method. Pan's contribution is particularly interesting because the Face Method does not simply modify a simplex pivoting rule. It proposes a different geometric philosophy for progressing through the feasible region. Earlier work by Zhang et al. [7] and Pan [9] also shows that the method has been subject to implementation development and technical discussion. Whether this approach will eventually yield substantial advantages across broad classes of practical LP problems remains an empirical question that deserves independent investigation rather than premature acceptance or rejection. The appropriate next step is therefore rigorous comparative benchmarking. Simplex, interior-point, and Face algorithms should be tested on common datasets covering small, medium, large, sparse, dense, degenerate, and ill-conditioned LP problems. Evaluations should report not only computational time but also iteration counts, numerical stability, memory requirements, scalability, warm-start performance, sensitivity-analysis capability, and solution interpretability.

The studies should disclose the software versions, hardware, parameter settings, stopping criteria, and datasets used so that the results can be reproduced. For scarce-resource allocation specifically, benchmarking should include realistic production-mix, transportation, workforce-allocation, agricultural, financial, energy, and supply-chain problems. Where possible, the tests should include changes in resource availability and objective-function coefficients to examine reoptimization and post-optimality analysis. An illustrative problem should

also be used to explain the geometric and computational differences among the methods. Such an example would clarify the procedures, while broader benchmark datasets would be needed to support claims about general performance. Only such evidence can determine whether moving face by face provides meaningful computational or managerial advantages over moving vertex by vertex. The present article is a conceptual discussion of the methods and the reasons for their differing levels of adoption. It does not report new computational experiments, and therefore does not claim to establish the comparative performance of the algorithms. Its purpose is to identify the historical, computational, and managerial questions that should guide further research.

Conclusion

The Simplex Method still overshadows the Face Method in scarce-resource allocation primarily because simplex possesses something that a developing method cannot acquire instantly: an accumulated institutional and computational history. Nearly eight decades of research have created textbooks, trained practitioners, optimized software, sensitivity-analysis procedures, theoretical extensions, and a vast body of applications around the simplex framework. The Face Method, by comparison, represents a developing alternative with a research history that predates Pan's 2025 monograph. Its face-to-face strategy offers an intellectually interesting alternative to conventional simplex geometry, and earlier implementation research provides a basis for further computational investigation. However, the available evidence discussed in this article is not sufficient to establish that the Face Method is generally faster, more robust, or more scalable than simplex across practical resource-allocation problems. Mathematical novelty and promising computational findings do not automatically translate into methodological displacement. The central issue, therefore, should not be framed as whether the Face Method has already 'defeated' the Simplex Method. A more productive question is whether independent theoretical and empirical research will establish classes of scarce-resource problems for which face-based algorithms offer measurable advantages in speed, robustness, scalability, interpretability, or reoptimization. The same comparisons should examine how effectively each method supports the sensitivity and post-optimality information required by decision-makers. For now, the Simplex Method remains the dominant language of scarce-resource allocation because it combines mathematical structure with an enormous stock of accumulated knowledge and implementation experience. The Face Method has developed into a substantial research direction, but it must continue to demonstrate the distance separating a promising algorithmic approach from a widely trusted decision-making technology. If future comparative evidence establishes consistent advantages for particular classes of problems, the role of the Face Method may expand. Until then, simplex's greatest competitive advantage may not be its age alone, but the sophisticated scientific and computational infrastructure that nearly eighty years of use has built around it.

Declaration: The author declares that generative artificial intelligence was used only for grammar and language enhancement in the preparation of this manuscript. The author takes responsibility for the content and accuracy of the work.

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